

THE USE OF MATHEMATICAL MORPHOLOGY IN IMAGE ENHANCEMENT

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ABSTRACT

This paper discusses several nonlinear filtering algorithms based on mathematical morphology. The structure and properties of these filters are discussed and examples of the typical performance of these filters are given.

1. INTRODUCTION

Image enhancement is a classical research topic in image processing. A central issue is the problem of effectively removing noise without smearing the geometrical features of the image. It is a well known fact that though linear filters are capable of effectively removing noise, they also blur geometrical features of the image. To address this problem, rank order and morphological filters have been developed [1-3].

The median filter, one type of rank order filter, has been extensively studied and used in a wide variety of applications. A great deal of work has been done investigating the deterministic and statistical properties of the median filter [4-6]. The most important property of this filter is its ability to preserve geometrical features in an image, especially edges and monotone areas, while removing impulsive noise [6]. The median filter does have limits in its performance. For example, if an image consists of very fine details such as thin lines and sharp corners, the median filter is likely to remove them [7]. There is also a lack of systematic design methodology for the median filter. One development in median filter design, known as the order statistic filter, was proposed by Bovik *et al.* [1]. The output of this filter is the linear combination of the ordered image data within a window. This filter was reported to share the advantageous properties of the averaging and median filters. The design of the filter employed the concept of combining linear and nonlinear operations. This concept has also been utilized in the design of other filtering algorithms [7,8].

Nieminen *et al.* proposed a class of filters that incorporate spatial information in a median filtering algorithm [9]. They reported improved detail preservation when compared with the simple median filter. Their approach was based on the multiple model idea in which filters are constructed to respond to different geometrical structures in the image. The multiple model concept has also been used in the design of linear filters such as the multiple model Kalman filter [10].

Morphological filters are nonlinear operations that exploit geometrical information in an image. Morphology was introduced by Serra and Matheron in the late 1960's [3]. Since then the methodology has been used extensively to address many problems in image processing and analysis [3].

In mathematical morphology an image is viewed as a set. Basic morphological transformations are performed using another set, known as a structuring element. The structuring element acts as a probe to exploit the geometrical structure in the image. The size and shape of the structuring element determine what geometrical features of the image are affected by the morphological transformation. Many morphological filtering algorithms have been developed [3,11,12].

In this paper, we discuss the structure and properties of several of these algorithms. The performance of the algorithms when applied to images corrupted with noise is presented. We also discuss several open research problems rela-

tive to these nonlinear filters.

2. MORPHOLOGICAL OPERATIONS

In this section we briefly review the basic morphological operations. Morphological operations can be divided into two classes: binary and grayscale. Binary operators consist of set operations and are applied to binary images. The binary morphological transformations of a binary image, A , by a binary structuring element, K , are defined by the following [3,13]:

$$\text{Erosion: } A \ominus K = \{z: K_z \subseteq A\} = \bigcap_{b \in K} A_{-b},$$

where A_b is the translate of A by an element b in the set K , and $-b$ is the inverse of b ;

$$\text{Dilation: } A \oplus K = \{z: K_z \cap A \neq \emptyset\} = \bigcup_{b \in K} A_{-b};$$

$$\text{Opening: } A \circ K = \{A \ominus K\} \oplus K;$$

$$\text{Closing: } A \bullet K = \{A \oplus K\} \ominus K.$$

Grayscale morphological transformations are extensions of binary morphological transformations [3,13,14]. The structuring element in this case is a grayscale function. Gray scale morphological operators can be defined as algebraic operations:

$$\text{Erosion: } (f \ominus k)(x) = \max_{z \in K} \{f(x+z) - k(z)\},$$

$$\text{Dilation: } (f \oplus k)(x) = \min_{z \in K} \{f(x-z) + k(z)\};$$

$$\text{Opening: } f \circ k = \{f \ominus k\} \oplus k;$$

$$\text{Closing: } f \bullet k = \{f \oplus k\} \ominus k.$$

where $k(z)$ is the grayscale structuring element function defined on the set K .

Among the many properties associated with the above operators, several are worth mentioning in this brief review. All four basic operations are *increasing*. That is, they preserve the order between sets or functions. This property is stated by the following: given sets A and B with order $A \supseteq B$, opening A and B each with the structuring element K will preserve the order, that is,

$$A \circ K \supseteq B \circ K.$$

Another property is *idempotency*. It is an invariance property for iterations of the operators. It can be described by the following: given a set A and a structuring element K , let $B = A \circ K$, then

$$B = B \circ K.$$

This property also holds for closing. The opening operation is also *antiextensive*, that is, the output of the opening operation is always contained in the original signal. As a dual operation of opening, the closing operation is *extensive*. That is, the output of the closing operation always contains the original signal. More detailed descriptions of both binary and grayscale morphological transformations can be found in [3,13].

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3. MORPHOLOGICAL FILTERS

Though both the opening and closing operations can be used to smooth an image, however, they perform the smoothing in different ways. Opening smooths a binary image by deleting from the image small features that cannot contain the structuring element. Opening smooths a grayscale image by cutting off narrow peaks into which the structuring element cannot fit. Closing fills narrow gaps between objects in a binary image, and fills narrow valleys in a grayscale image. A traditional morphological filter usually consists of an opening and a closing operation, applied in series [14,16]. The most common support regions used for structuring elements are the circle and the square. If information is known about the geometrical structure of the original image, then the structuring element can be designed to preserve this geometrical structure. The structuring element can also be specifically designed to remove noise whose geometrical structure is known. Sternberg used this approach to construct a pair of structuring elements to remove streaks appearing in gel images [15]. Special structuring elements have also been designed to suppress impulsive noise and normalize the background of digitized EKG data and for target enhancement [16,17].

Applying opening and closing in a different order usually produces very different results. This situation, known as *bias*, is caused by the extensiveness properties of opening and closing [3,11,18]. Figure 1 illustrates such a situation for a one dimensional signal. In this example, applying the opening first, then closing, produces a result most of which is less than the original signal. If closing is applied first, followed by opening, the result will be greater and will appear brighter in the image. One way to minimize the bias is to apply morphological filters sequentially with structuring elements of increasing size [3,11]. Another way is to average the results of the opening and closing [12,16].

The performance of the previous morphological filtering algorithms is quite satisfactory when the image only contains details larger than the size of the structuring element. When an image contains very fine detail it is necessary to suppress the noise without blurring or removing the fine image detail. Unfortunately, the use of one structuring element will seldom meet this requirement. A small structuring element will not remove the noise effectively, a larger structuring element will eliminate fine details along with noise.

To improve the performance, a morphological filter using multiple structuring elements was proposed by Stevenson and Arce [18]. We refer to this as the 2D CO filter (see Figure 2). The filter consists of a cascade of two stages each of which consists of multiple morphological operators of one type. In each stage, multiple operators are applied to the input image using different structuring elements. The output of each stage is selected according to the morphological operator used: maximum for outputs of the openings, or minimum for outputs of the closings. If we assume that the first stage consists of closing operators and the second stage of opening operators, the algorithm is described below. For an input image, f , and a set of structuring elements, $\{k_1, k_2, \dots, k_n\}$, the output y of the first stage is given by

$$y = \min \{f \bullet k_1, f \bullet k_2, \dots, f \bullet k_n\} \quad (1)$$

The output z of the second stage is given by

$$z = \max \{y \circ k_1, y \circ k_2, \dots, y \circ k_n\} \quad (2)$$

The purpose of each stage is to preserve the geometrical features in the image that match any one of the given structuring elements. For example, four structuring elements consisting of lines with different directions were used in [18]. This results in all lines in the image with direction matching the structuring elements and with lengths no shorter than the structuring elements being preserved. It has also been shown that the idempotent property holds for this filter [18,19].

The morphological filters we have discussed so far are suitable for removing impulsive noise, that is, noise that consists of isolated clusters of pixels whose values are either much higher or lower than values of neighboring image pixels. By designing the size of the structuring element to be larger than

that of the noise cluster, the impulsive noise is removed by opening and closing operations. These filters do not perform well on an image that is corrupted by non-impulsive noise. The desire of combining the properties of linear filtering to suppress non-impulsive noise and that of morphological filtering to preserve geometrical features led to the introduction of a generalized morphological filter [12]. The filter is described below.

For the input image, f , and structuring elements, $\{k_1, k_2, \dots, k_n\}$, the output of the first stage of the filter is given by

$$y = \sum_{i=1}^n \alpha_i y_{(i)} \quad (3)$$

where $y_{(i)}$ are the increasingly ordered outputs of $f \bullet k_j$. The α_i are real constants with $\sum_{i=1}^n \alpha_i = 1$. The output of the second stage is given by

$$z = \sum_{i=1}^n \beta_i z_{(i)} \quad (4)$$

where $z_{(i)}$ are the increasingly ordered outputs of $y \circ k_j$. The β_i are real constants with $\sum_{i=1}^n \beta_i = 1$. A block diagram of the filter is given in Figure 3.

The values of the real constants α_i and β_i are determined by the particular application. For example, by letting $\beta_i = \alpha_i = \frac{1}{n}$, $\forall i$, an averaging version of the filter is obtained. Since the outputs of morphological operators contain structural information, the averaging process applied to these outputs will result in reduced blurring of geometrical image features. For $\beta_1 = 1$, $\beta_i = 0$ for $i \neq 1$, and $\alpha_n = 1$, $\alpha_i = 0$ for $i \neq n$, this filter is identical to the 2D CO filter.

Bias also exists for the two filters discussed above. Since the sizes of the multiple structuring elements used are usually smaller than the single structuring element used in traditional morphological filters, the bias problem for the filters is not as significant as with the traditional filter.

4. EXPERIMENTS

The morphological filters described above were tested using two noisy images. The traditional (closing-opening) morphological filter used a 3x3 square structuring element. Figure 4 shows the four structuring elements used by the filters using multiple structuring elements. The image in Figure 5 was corrupted by impulsive noise, and the image in Figure 6 was corrupted by Gaussian noise. The results in Figure 5 clearly show that the 2D CO morphological filter exhibits a significant improvement in the performance when compared with more classical morphological filters. The 2D CO filter is not only able to effectively remove impulsive noise, but also preserve very fine detail in the image. In this example any line that is not less than three pixels in length is preserved.

The results in Figure 6 also show the improvement in the performance of the new filter when applied to the image corrupted by non-impulsive noise. We can see that the averaging version of the generalized morphological filter produces much sharper edges and smoother constant areas than the traditional morphological filter.

5. OPEN RESEARCH PROBLEMS

While several interesting and useful morphological filtering structures have been developed, there does not exist a sound theoretical basis for the development and analysis of new filters. An open problem for general nonlinear filter analysis is how to characterize a filter's behavior with regard to the recovery of a signal from noise corrupted data. The analysis is generally divided into two parts. First, the syntactic or geometrical properties of a filter are studied by examining the effect of the filter on structure in the original signal. Secondly, statistical properties are derived by assuming an input signal with known statistics and determining the output statistics of the filter. Although this does not provide a com-

plete characterization of the filter, this analysis provides valuable insight into a filter's behavior.

The syntactic performance of a morphological filter depends mainly on the size and shape of its structuring elements. In most cases, the geometrical structure of the original signal and of the noise are hard to characterize. In such cases, the design of the structuring elements for morphological filtering is based on trial and error [16]. Serra [3] provides an in depth analysis into some of these theoretical issues, however, his focus is largely on the properties of set operations related to morphological filters. The performance of morphological filters used for noise removal is not discussed. Presently, much theoretical analysis remains to be done relating the effectiveness of noise suppression with the design of the structuring element.

The statistical analysis of morphological filters has been proven to be very difficult. Stevenson and Arce derived output distributions for a simple morphological filter, however, the analysis is not easily extended to more general cases [18]. One of the main difficulties in statistical analysis is the signal correlations induced by the morphological transformations. The relationships between morphological filters and rank order filters have been studied by Maragos and Schafer [20]. They have shown that rank order filters form a subset of morphological filters. Furthermore, they discovered that the opening-closing and closing-opening can yield the root signal of a median filter in one pass, and are computationally less expensive than the median filter. The establishment of these relationships provides researchers in the area of mathematical morphology an opportunity to apply the rich theoretical work on rank order filters to morphological filters.

While the understanding and characterization of the properties of a well defined morphological filtering structure is very important, the reverse problem of finding a filter structure to meet a set of specifications is an equally important problem. The synthesis of a rank order filter has been carried out using the more general concept of a stack filter [21]. This theoretical framework has made it possible to design the optimal stack filter [22]. Since the concept of threshold decomposition and the stacking property are also applicable to morphological filters [21], stack filters may be useful in the theoretical analysis of morphological filters. Other well established theoretical results for stack filters may also be applied to develop the theoretical framework for morphological filters.

While morphological filters have demonstrated the strong ability to suppress noise and preserve geometrical signal structure, much work remains in the theoretical analysis of these filters. In order for more advanced morphological filters to be developed, we must be able to characterize the effect of a known filter and we must be able to synthesize a filter structure for a given specification.

6. CONCLUSION

In this paper several morphological filters were discussed. The results clearly illustrate recent improvements in the performance of morphological filters. Much work needs to be done on the systematic design and analysis of more advanced morphological filter structures.

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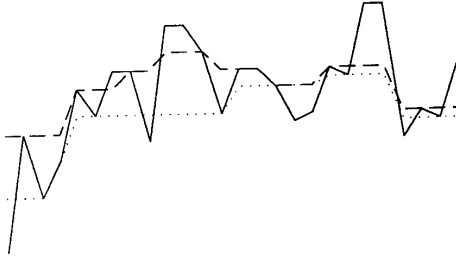


Figure 1. Illustration of the bias problem. The structuring element used in this example is a line of three samples in length. The solid line represents the original data, the dotted line represents the result of the opening-closing operation, and the dashed line represents the result of the closing-opening operation.

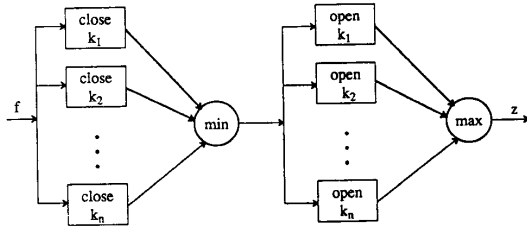


Figure 2. Block diagram of the 2D CO filter.

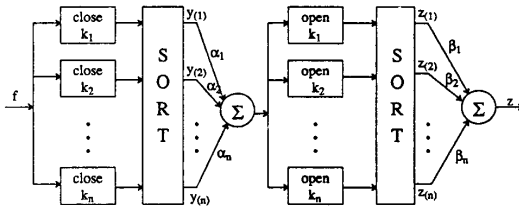


Figure 3. Block diagram of the generalized morphological filter.



Figure 4. Illustration of the four structuring elements used by the filters in Figure 5 and Figure 6.



Figure 5. Upper left: original image; Upper right: corrupted image with impulsive noise; Lower left: resultant image using a closing-opening morphological filter with a 3x3 square structuring element; Lower right: resultant image of 2D CO filter using the structuring elements in Figure 4.

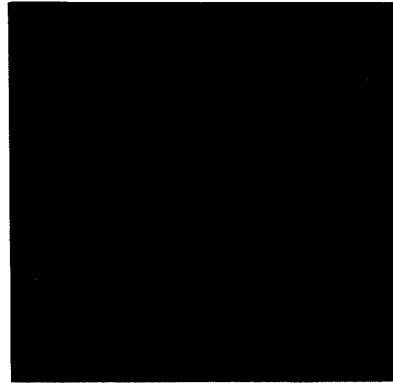


Figure 6. Upper left: original image; Upper right: corrupted image with Gaussian noise; Lower left: resultant image using a closing-opening morphological filter with a 3x3 square structuring element; Lower right: resultant image of the averaging version of the generalized morphological filter using the structuring elements in Figure 4;